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A NITSCHKE FINITE ELEMENT METHOD FOR DYNAMIC CONTACT

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We present a new approximation of elastodynamic frictionless contact problems based both on the finite element method and on an adaptation of Nitsche's method which was initially designed for Dirichlet's condition [4]. The corresponding space semi-discretized weak form reads:

$$\left\{ \begin{array}{l} \text{Find a displacement } \mathbf{u}^h : [0, T] \rightarrow \mathbf{V}^h \text{ such that for } t \in [0, T] : \\ \langle \rho \ddot{\mathbf{u}}^h(t), \mathbf{v}^h \rangle + A_{\Theta\gamma_h}(\mathbf{u}^h(t), \mathbf{v}^h) + \int_{\Gamma_C} \frac{1}{\gamma_h} [P_{\gamma_h}(\mathbf{u}^h(t))]_+ P_{\Theta\gamma_h}(\mathbf{v}^h) d\Gamma = L(t)(\mathbf{v}^h), \\ \forall \mathbf{v}^h \in \mathbf{V}^h, \\ \mathbf{u}^h(0, \cdot) = \mathbf{u}_0^h, \quad \dot{\mathbf{u}}^h(0, \cdot) = \dot{\mathbf{u}}_0^h. \end{array} \right. \quad (1)$$

In the above formulation \mathbf{V}^h is a finite element space built from standard Lagrange finite elements, piecewise linear or quadratic, $A_{\Theta\gamma_h}(\mathbf{u}^h, \mathbf{v}^h) := \int_{\Omega} \boldsymbol{\sigma}(\mathbf{u}^h) : \boldsymbol{\varepsilon}(\mathbf{v}^h) d\Omega - \int_{\Gamma_C} \Theta\gamma_h \sigma_n(\mathbf{u}^h) \sigma_n(\mathbf{v}^h) d\Gamma$ and $P_{\Theta\gamma_h}(\mathbf{v}^h) := v_n^h - \Theta\gamma_h \sigma_n(\mathbf{v}^h)$. The linear form $L(\cdot)$ stands for prescribed body and boundary forces. The domain of the elastic body is denoted by Ω and the contact boundary by Γ_C ; T is the final time of simulation; ρ is the density of the elastic material; the notation v_n^h stands for the normal component on Γ_C of \mathbf{v}^h , and $\sigma_n(\mathbf{v}^h)$ is the normal stress on Γ_C ; \mathbf{u}_0^h (resp. $\dot{\mathbf{u}}_0^h$) is an approximation of the initial displacement \mathbf{u}_0 (resp. the initial velocity $\dot{\mathbf{u}}_0$). The notation $[\cdot]_+$ stands for the positive part of a scalar quantity, and $\langle \cdot, \cdot \rangle$ stands for the $L^2(\Omega)$ inner product. The parameter γ_h

is a positive piecewise constant function on the contact interface Γ_C ($\gamma_h(x) = \gamma_0 h_T(x)$, where γ_0 is a positive constant and $h_T(x)$ is the size of the mesh element T). Note the additional numerical parameter Θ which can be freely chosen in \mathbb{R} . As in Nitsche's method for (static) unilateral contact, values of interest for Θ are $-1, 0, 1$ [1].

A main interesting characteristic is that this approximation produces well-posed space semi-discretizations, for any value of $\gamma_0 > 0$, contrary to standard finite element discretizations. We study associated energy conservation properties and manage to prove that :

$$\frac{d}{dt} E_{\Theta}^h(t) = (\Theta - 1) \int_{\Gamma_C} \frac{1}{\gamma_h} [P_{\gamma_h}(\mathbf{u}^h(t))]_+ \dot{u}_n^h(t) d\Gamma$$

where $E_{\Theta}^h(t) := E^h(t) - \Theta R^h(t)$ is an augmented energy associated to the semi-discrete solution $\mathbf{u}^h(t)$. The mechanical energy is given by $E^h(t) := \frac{1}{2} [\rho \|\dot{\mathbf{u}}^h(t)\|_{0,\Omega}^2 + \frac{1}{2} a(\mathbf{u}^h(t), \mathbf{u}^h(t))]$ and $R^h(t) := \frac{1}{2} \left[\|\gamma_h^{\frac{1}{2}} \sigma_n(\mathbf{u}^h(t))\|_{0,\Gamma_C}^2 - \|\gamma_h^{-\frac{1}{2}} [P_{\gamma_h}(\mathbf{u}^h(t))]_+\|_{0,\Gamma_C}^2 \right]$ is an extra term, which represents, roughly speaking, the non-fulfillment of the contact conditions at the semi-discrete level. Note in particular that the symmetric variant ($\Theta = 1$) conserves the discrete augmented energy E_{Θ}^h .

Various time-discretizations for (1) are then considered, with the families of θ -schemes and Newmark schemes. We also introduce a new hybrid scheme which is second-order accurate and numerically stable without any restriction on the time-step in the case $\Theta = 1$. This new scheme is inspired from [2, 3], and introduces much less dissipation than unconditionally stable variants of θ -schemes and Newmark schemes. We study theoretically the well-posedness of each discrete scheme as well as its energy conservation properties. We finally achieve the corresponding numerical experiments.

REFERENCES

- [1] F. CHOULY, P. HILD, AND Y. RENARD, *Symmetric and non-symmetric variants of Nitsche's method for contact problems in elasticity: theory and numerical experiments*. <http://hal.archives-ouvertes.fr/hal-00776619>. To appear in Math. Comp.
- [2] O. GONZALEZ, *Exact energy and momentum conserving algorithms for general models in nonlinear elasticity*, Comput. Methods Appl. Mech. Engrg., 190 (2000), pp. 1763–1783.
- [3] P. HAURET AND P. LE TALLEC, *Energy-controlling time integration methods for nonlinear elastodynamics and low-velocity impact*, Comput. Methods Appl. Mech. Engrg., 195 (2006), pp. 4890–4916.
- [4] J. NITSCHKE, *Über ein Variationsprinzip zur Lösung von Dirichlet-Problemen bei Verwendung von Teilräumen, die keinen Randbedingungen unterworfen sind*, Abhandlungen aus dem Mathematischen Seminar der Universität Hamburg, 36 (1971), pp. 9–15.